THE General Radio EXPERIMENTER

ELECTRICAL MEASUREMENTS AND THEIR INDUSTRIAL APPLICATIONS

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Also

IN THIS ISSUE

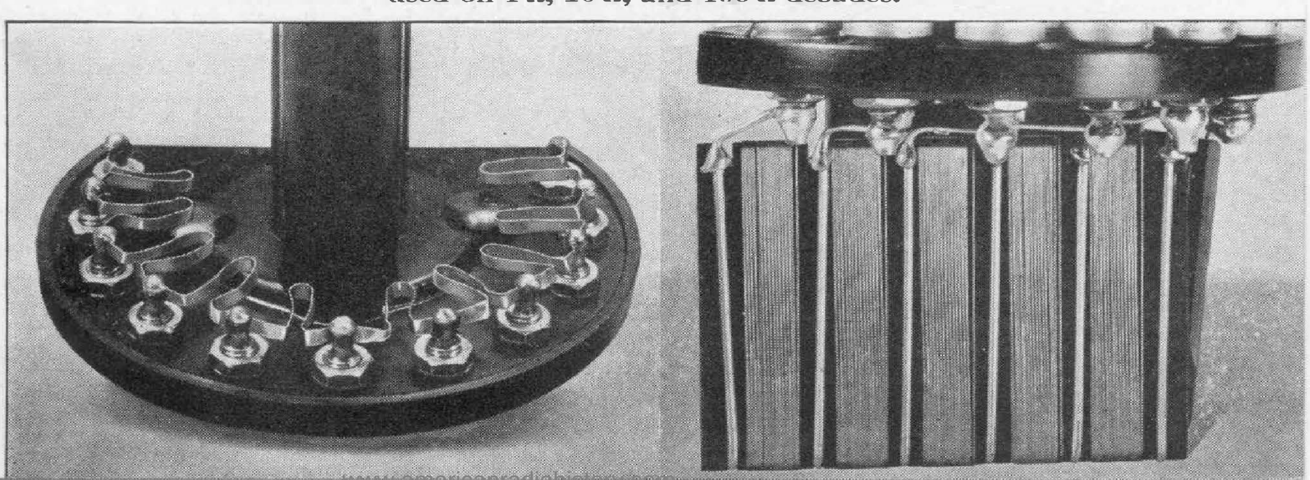
RADIO-FREQUENCY CHARACTERISTICS OF DECADE RESISTORS

● FOR GENERAL MEASUREMENTS at commercial and audio frequencies, decade resistors have been found invaluable laboratory tools. In these low-frequency applications their reactance components are usually negligible, and they consequently approach closely the ideal resistance standards desired for lumped-parameter circuits.

As the frequency increases, their reactance components become comparable with the resistance components, and the residual inductance and capacitance combine to change the effective terminal resistance from its low-frequency value. This change in resistance is generally augmented by skin-effect, and the general characteristics of the resistor deviate more and more widely from the ideal.

For many applications, however, the deviations from the ideal are not sufficiently important to offset the convenience and flexibility that are characteristic of decade resistors. The data contained in this article are presented to enable users of General Radio decade resistors to estimate the degree of departure from the ideal and to make corresponding corrections. They were taken with an experimental

FIGURE 1. (*Left*) Bifilar winding used on 0.1 Ω decades. (*Right*) Ayrton-Perry winding used on 1 Ω, 10 Ω, and 100 Ω decades.



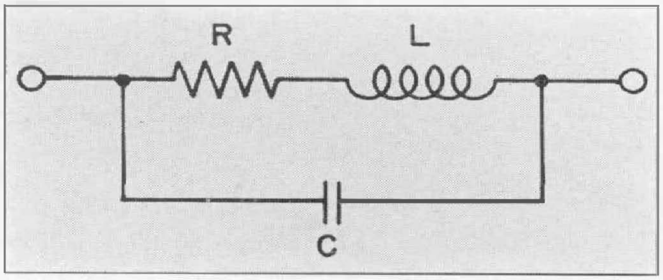


FIGURE 2. Equivalent circuit of a resistor having residual inductance and capacitance.

model of a new radio-frequency bridge[1] and, because of the improved technique that has been developed, are more accurate than previously published data.[2] This is particularly true of the resistance values, which formerly were extrapolated from low-frequency measurements and which are now measured directly at the high frequencies.

ANALYSIS

The approximate equivalent circuit commonly used to represent a resistor having residual inductance and capacitance is shown in Figure 2.

The expressions for the effective terminal resistance, R_e, and reactance, X_e, of this circuit are:[3]

$$R_e = \frac{R}{(1 - \omega^2 LC)^2 + (\omega CR)^2} \quad (1)$$

$$X_e = \frac{\omega[L(1 - \omega^2 LC) - CR^2]}{(1 - \omega^2 LC)^2 + (\omega CR)^2} \quad (2)$$

At low frequencies, where the terms in ω^2 are negligibly small, the effective resistance, R_e, is simply equal to the nominal resistance, R. The effective reactance, X_e, is equal to that of a constant effective inductance $[L - CR^2]$. For low resistance values the CR^2 term is negligible and the reactance positive; for high resistance values it is predominant and the reactance negative. In either case the reactance varies directly as the frequency.

At high frequencies, the terms in ω^2 become important and both the effective resistance and the effective inductance vary with frequency. For low resistance values the $(\omega CR)^2$ term is negligible and the effective resistance and inductance increase with frequency because of the resonance between L and C; for high resistance values the $(\omega CR)^2$ term is

[1] D. B. Sinclair, "A Radio-Frequency Bridge for Measurements at Frequencies from 400 kc to 60 Mc," *Proc. I. R. E.*, Vol. 28, No. 10, Nov. (1940).

[2] R. F. Field, "Frequency Characteristics," General Radio *Experimenter*, Vol. 6, No. 9, p. 1; Feb. (1932).

[3] These expressions are presented in normalized form in the General Radio *Experimenter*, Vol. 8, No. 8, p. 6; Jan. (1939) with families of curves in terms of the parameter $D_0 = R/\sqrt{\frac{L}{C}}$ and variable $\omega/\omega_0 = \omega\sqrt{LC}$. Because of the wide variation in D_0 and ω_0 in decade resistors, however, the expressions are more convenient in the form shown here.

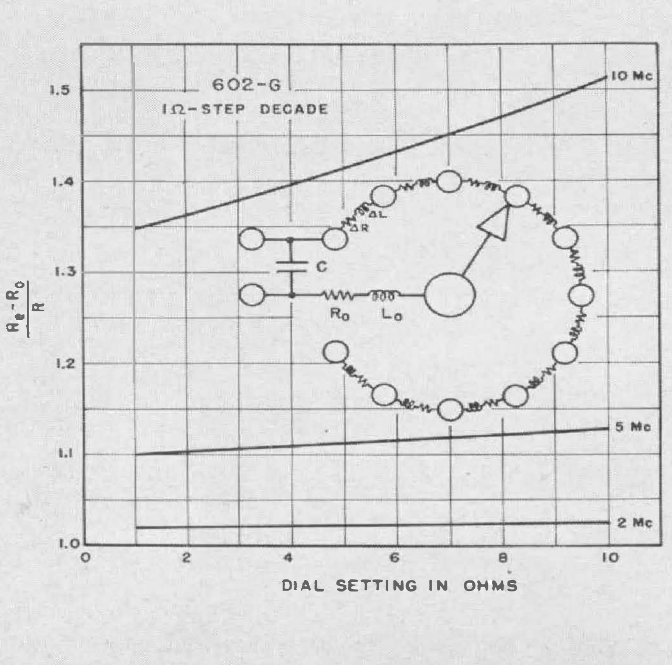


FIGURE 3. Plot of change in resistance of 1 Ω-step decade of TYPE 602-G Decade Resistance Box at frequencies of 2, 5, and 10 Mc. The values plotted refer to the change in resistance from zero se ting. To obtain the absolute resistance, the zero resistance, R_0, must be added. The percentage increase varies approximately as the square of the frequency; the zero resistance varies approximately as the square root of the frequency. In the approximate equivalent circuit shown:

$\Delta R = 1\ \Omega$ (nominal)
$L_0 = 310 \times 10^{-9}$ h
$\Delta L = 52 \times 10^{-9}$ h per step
$C = 27 \mu\mu f$
$R_0 = 0.05\ \Omega$ at 2 Mc
$0.09\ \Omega$ at 5 Mc
$0.013\ \Omega$ at 10 Mc

The skin-effect in the resistance cards of this decade is approximately:

1.6% at 2 Mc
8.8% at 5 Mc
25% at 10 Mc

predominant, and the resistance and inductance decrease with frequency because of the shunting effect of C.

Proper proportioning of the residual parameters, L and C, for a given resistance value, will lead to the best design for a single fixed resistor and even to the best compromise design for a line of fixed resistors.[4] For a decade resistor, however, in which the resistance range is relatively large, the effect of the residual parameters can only be minimized by making the residual parameters themselves as small as possible.

TYPE 602-G DECADE-RESISTANCE BOX

At radio frequencies the resistance values most commonly encountered usually lie in the range from 1 Ω to 1000 Ω. The Type 602-G Decade-Resistance Box, which covers this range in three decades, was therefore chosen as representative.

In Figure 3 is shown the variation in resistance of the 1 Ω-step decade at frequencies of 2, 5, and 10 Mc. For convenience, the resistance change from zero dial setting to any other dial setting is expressed as a fraction of the nominal resistance at the final dial setting. The plot is therefore really of incremental resistance and is particularly useful when the resistor is left in circuit at all times and is varied to alter circuit conditions. The zero resistance, R_0, which arises in the wiring, must be added to the value indicated in Figure 3 to obtain the absolute terminal resistance at any setting.

The behavior of the equivalent circuit shown, with the constants listed in the caption, approximates closely that of the actual decade, both for resistance and reactance.

The reactance, for all practical purposes, is that of an inductance equal to the sum of the zero inductance, L_0, which arises in the wiring, and the inductance of the resistance cards. The inductance of a card, ΔL, is essentially independent of its location in the decade and is therefore listed as a constant value. For this decade, variations in the capacitance, C, could not be accurately deduced from the observed measurements and it is therefore assumed independent of switch setting.

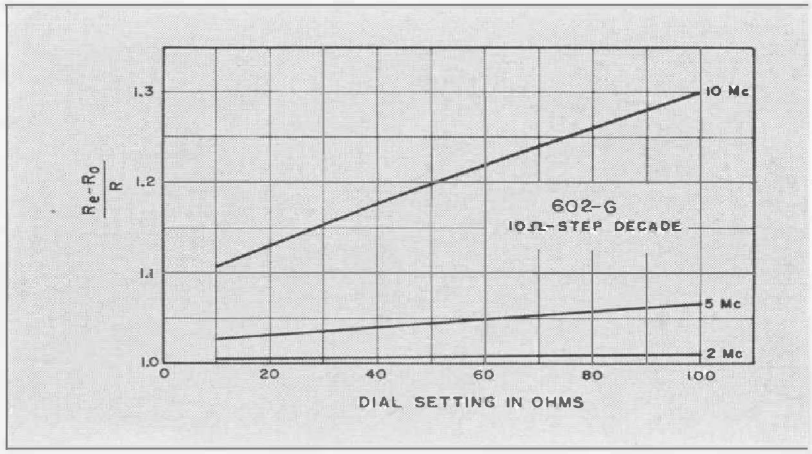


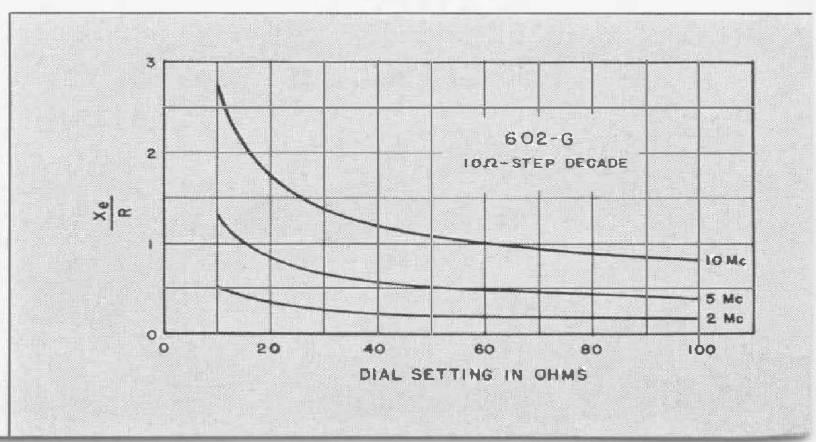
Figure 4. Plot of change in resistance of 10 Ω-step decade of Type 602-G Decade Resistance Box at frequencies of 2, 5, and 10 Mc:

$$\Delta R = 10\ \Omega \text{ (nominal)}$$
$$L_0 = 310 \times 10^{-9}\ \text{h}$$
$$\Delta L = 110 \times 10^{-9}\ \text{h per step}$$
$$C = 26.0 - 23.5\ \mu\mu\text{f}$$
$$R_0 = 0.05\ \Omega \text{ at 2 Mc}$$
$$0.09\ \Omega \text{ at 5 Mc}$$
$$0.13\ \Omega \text{ at 10 Mc}$$

The skin-effect in the resistance cards of this decade is approximately:

0.0% at 2 Mc
0.3% at 5 Mc
1.2% at 10 Mc

Figure 5. Plot of ratio of reactance to nominal resistance of 10 Ω-step decade of Type 602-G Decade Resistance Box at frequencies of 2, 5, and 10 Mc.



[4]See D. B. Sinclair, "The Type 663 Resistor — a Standard for Use at High Frequencies," General Radio *Experimenter*, Vol. 8, No. 8, p. 6; Jan. (1939). [Footnote 3.]

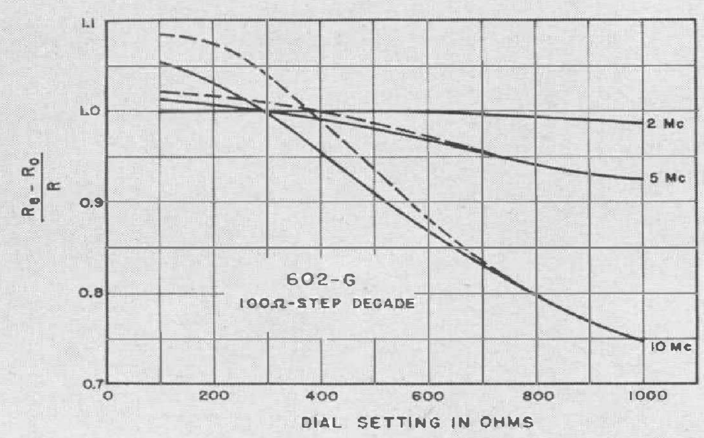


FIGURE 6. Plot of change in resistance of 100 Ω-step decade of TYPE 602-G Decade Resistance Box at frequencies of 2, 5, and 10 Mc. The solid curves represent measured values. The dotted curves are based on the following constants:

$\Delta R = 100\ \Omega$ (nominal)
$L_0 = 310 \times 10^{-9}$ h
$\Delta L = 287 \times 10^{-9}$ h per step
$C = 21.0 - 12.7\ \mu\mu f$
$R_0 = 0.05\ \Omega$ at 2 Mc
$0.09\ \Omega$ at 5 Mc
$0.13\ \Omega$ at 10 Mc

The skin-effect in the cards of this decade is negligible.

The rise in resistance at a setting of 1 Ω is largely caused by skin-effect, the further rise as the resistance is increased being caused by resonance between the residual inductance and capacitance.

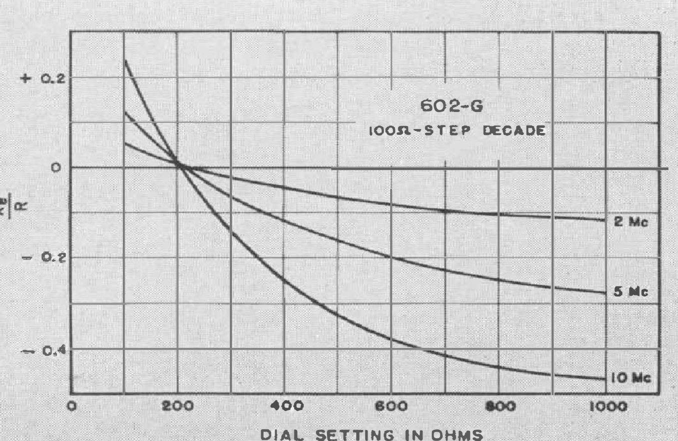
In Figures 4 and 5 are shown curves of effective resistance and reactance for the 10 Ω-step decade. The skin-effect for this decade is much less than for the 1 Ω decade, and the resistance rise shown is largely caused by resonance at all settings. The relatively large rise at the 10 Ω setting occurs because the zero inductance in the wiring is considerably larger than that of the card itself.

For convenience, the effective reactance is expressed as a fraction of the corresponding nominal resistance. The reason that the reactance curves rise at the low-resistance settings is again that the zero inductance in the wiring is large compared with the inductance of a card and so contributes proportionately more to the over-all inductance at low settings.

The listed circuit constants, when used in the equivalent circuit of Figure 3, yield a close approximation to the observed measurements of both resistance and reactance. To a first approximation the effective capacitance, for this decade, is found to vary linearly, as a function of dial setting, from 26.0 $\mu\mu$f, at a setting of 10 Ω, to 23.5 $\mu\mu$f, at a setting of 100 Ω. This comes about because capacitance from the resistance cards to ground is less effective when they are active than when they are out of circuit.[5] As the resistance increases, therefore, the capacitance decreases. It is lower, in general, than it is for the 1 Ω-step decade, because any capacitance to ground in that decade, which is between the 10 Ω-step decade and ground, is short-circuited.

In Figures 6 and 7 are shown curves of effective resistance and reactance for the 100 Ω-step decade. The resistance, for low settings, rises above the d-c value because of resonance. As the resistance increases, however, the terms containing R in Equation (1) become predominant, and the resistance drops because of

FIGURE 7. Plot of ratio of reactance to resistance of 100 Ω-step decade of TYPE 602-G Decade Resistance Box at frequencies of 2, 5, and 10 Mc.

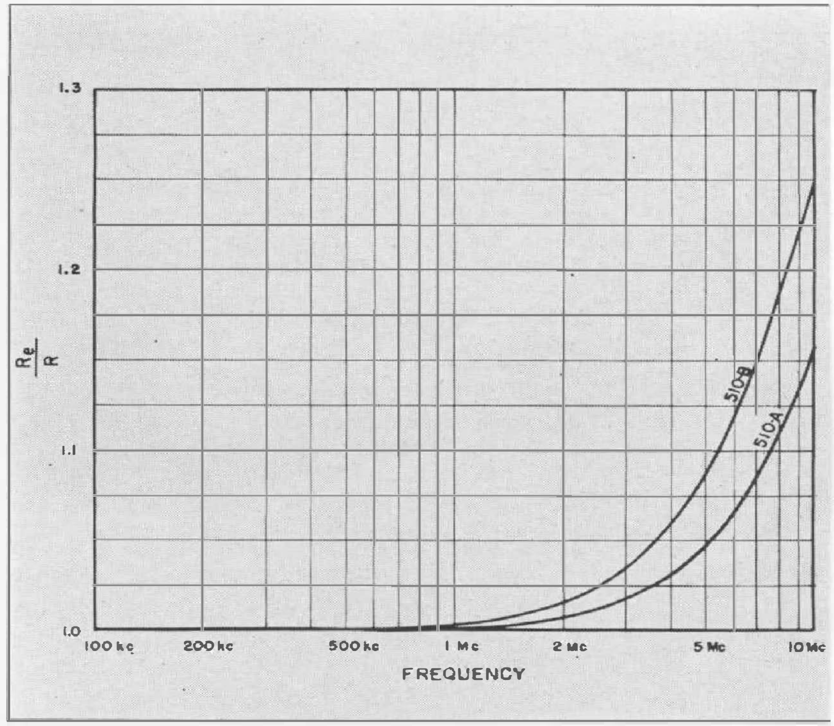


[5]Because of the voltage distribution, the effective capacitance of the cards in circuit to ground is of the order of 1/3 of their total capacitance to ground.

Figure 8 (*right*). Resistance rise of Type 510 Decade Resistance Units caused by skin-effect. For the 0.1 Ω-step unit (Type 510-A) and the 1 Ω-step unit (Type 510-B) the variation of resistance with frequency is almost entirely caused by skin-effect and is consequently independent of dial setting. The equivalent circuit constants are:

	510-A	510-B
ΔR	0.1 Ω	1 Ω (nominal)
L_0	22.6×10^{-9} h	22.6×10^{-9} h
ΔL	13.7×10^{-9} h	55.6×10^{-9} h

The zero resistance, R_0, for these units is about 0.01 Ω at 10 Mc.



the shunting effect of *C*. For the same reason the reactance, which is positive for low values of resistance, becomes negative for high values. The skin-effect for the resistance cards of this decade is negligible.

The values of the listed circuit constants, when applied to the equivalent circuit of Figure 3, give close agreement with the measured reactance and with the measured resistance at high-resistance settings. At low-resistance settings, however, the computed resistance deviates from the measured resistance as shown in the dotted curve of Figure 6. This deviation arises because the displacement currents to ground from the inactive cards in the 100 Ω-step decade pass first through the resistances of these cards as conduction currents. The consequent power loss is relatively great and causes a resistance shunting effect that decreases the effective resistance from the value it would have if the capacitance were loss-free.[6] The effect is a maximum at resistance settings of 200 and 300 ohms. For higher resistances the discrepancy decreases because the inactive cards become fewer; for lower resistances the discrepancy decreases because the resistance goes down more rapidly than the effective shunting resistance.[7]

Figure 9 (*below*). Plot of change in resistance of 10 Ω-step unit (Type 510-C) at frequencies of 5 and 10 Mc:

$$\Delta R = 10\ \Omega \text{ (nominal)}$$
$$L_0 = 22.6 \times 10^{-9}\ \text{h}$$
$$\Delta L = 110 \times 10^{-9}\ \text{h}$$
$$C = 7.7 - 4.5\ \mu\mu\text{f}$$
$$R_0 = 0.004\ \Omega \text{ at 2 Mc}$$
$$0.007\ \Omega \text{ at 5 Mc}$$
$$0.004\ \Omega \text{ at 10 Mc}$$

The skin-effect in the resistance cards of this decade is approximately:

0.0% at 2 Mc
0.3% at 5 Mc
1.2% at 10 Mc

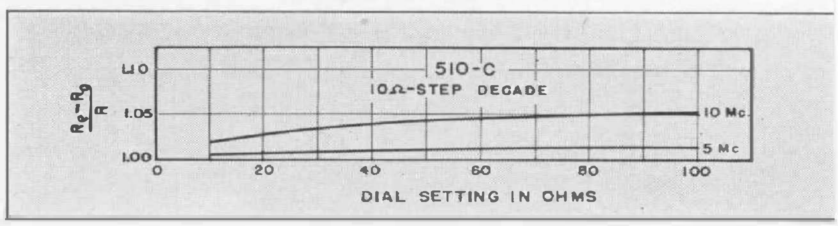
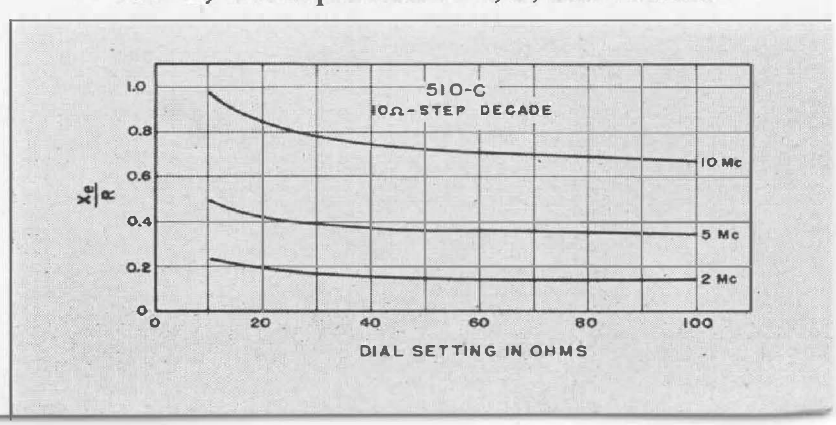


Figure 10 (*below*). Plot of ratio of reactance to nominal resistance of 10 Ω-step unit (Type 510-C) at frequencies of 2, 5, and 10 Mc.



[6]See, for instance, Jones & Josephs, *Journal American Chemical Society*, Vol. 50, p. 1049, 1928, for a discussion of this "hung on" effect at low frequencies.

[7]Some discrepancy between computed and measured resistance would still be expected at the high-resistance settings of the 10 Ω-step decade. That this was not explicitly noted is ascribed to the necessarily approximate nature of the equivalent circuit and the method of deducing the values of *L* and *C* for use with it.

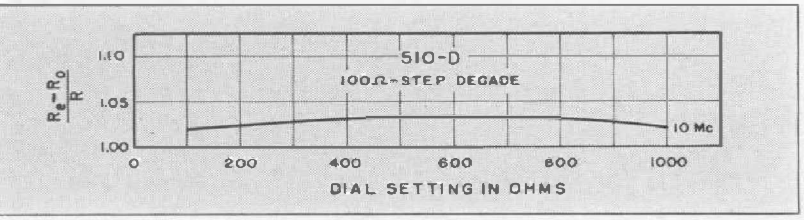


FIGURE 11. Plot of change in resistance of 100 Ω-step unit (TYPE 510-D) at a frequency of 10 Mc:

$$\begin{aligned} \Delta R &= 100\ \Omega\ \text{(nominal)} \\ L_0 &= 22.6 \times 10^{-9}\ \text{h} \\ \Delta R &= 287 \times 10^{-9}\ \text{h} \\ C &= 7.7 - 4.5\ \mu\mu\text{f} \\ R_0 &= 0.004\ \Omega\ \text{at 2 Mc} \\ &\ 0.007\ \Omega\ \text{at 5 Mc} \\ &\ 0.01\ \Omega\ \text{at 10 Mc} \end{aligned}$$

The skin-effect in the resistance cards of this decade is negligible.

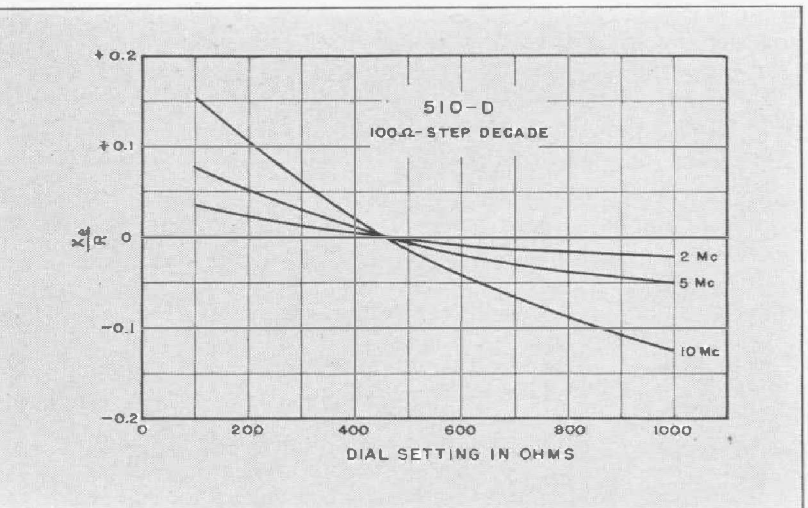


FIGURE 12. Plot of ratio of reactance to nominal resistance of 100 Ω-step unit (TYPE 510-D) at frequencies of 2, 5, and 10 Mc.

It should be emphasized that when resistance in more than one decade is being used, the plotted results cannot be combined directly because of the interaction between the residual parameters in the various decades. To a first approximation, however, the sum of the reactances can be assumed equal to the total reactance, and the multiplying factor for the resistance setting in the higher decade can be applied to the sum of the resistance settings.

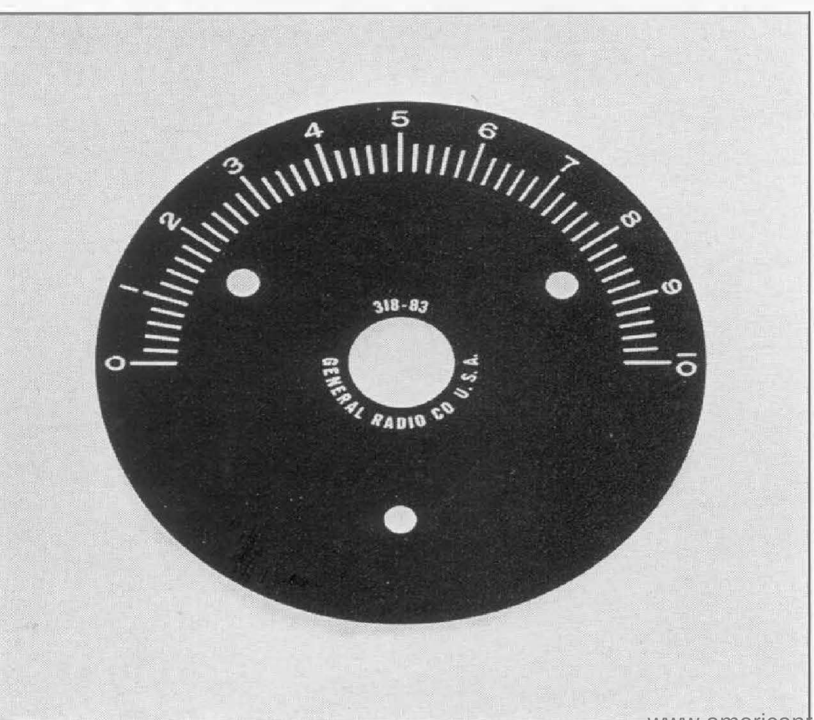
The variations in resistance and reactance of the TYPE 510 Decade-Resistance Units that are used in the TYPE 602 Decade-Resistance Boxes are plotted in Figures 8 to 12. In general the variations follow the same pattern as those for the corresponding decades in the decade-resistance boxes, but the variations in resistance and the magnitudes of the reactance are less because the capacitance and wiring inductance are much reduced. They were measured without shield cans in order to make this reduction of capacitance as great as possible.

—D. B. SINCLAIR

NEW DIAL PLATE

● **THE NEW** TYPE 318-C Dial Plate, shown here, is intended for use with air condensers, variometers, and other variable circuit elements having a 180-degree angle of rotation. The scale progresses in a counterclockwise direction and has ten main divisions, each of which is divided into five small divisions.

The scale is photo-etched with raised



View of the TYPE 318-C Dial Plate.

nickel-silver graduations on a flat black background. The diameter of the plate is three inches. It can be used with a 1⅝-inch knob, either skirted or with pointer. The center hole will clear a ⅜-inch shaft.

Net Weight: ¾ ounce.

Type		*Price*
318-C	....	$0.35

USING THE VARIAC AT LOW FREQUENCIES

● **ALTHOUGH VARIACS** are designed for 60-cycle service, they can be used at lower frequencies provided the ratings are reduced.

Standard ratings are for 60 cycles, but an adequate margin is allowed so that the full 60-cycle rating can be obtained at 50 cycles. For frequencies lower than 50 cycles the voltage applied should be less than the rated voltage of the Variac in the same ratio as the reduction in frequency. For instance a 230-volt, 60-cycle Variac can be used on a 115-volt, 25-cycle line. The current rating remains unchanged and the volt-ampere rating is consequently halved.

Used at 25 cycles, the ratings for the 230-volt, 60-cycle Variacs are as shown in the table below.

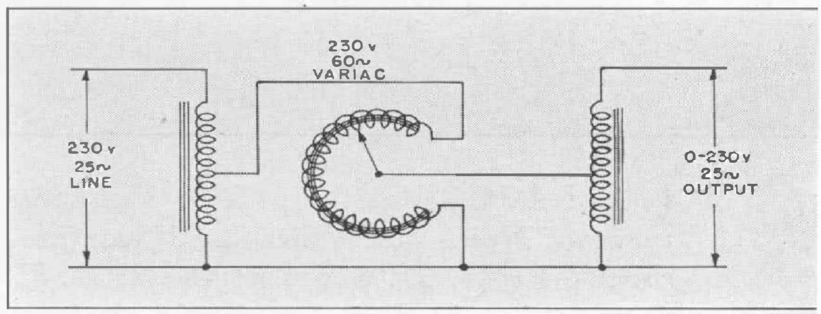


FIGURE 1. Showing how autotransformers are used with a Variac on 230-volt, 25-cycle circuits.

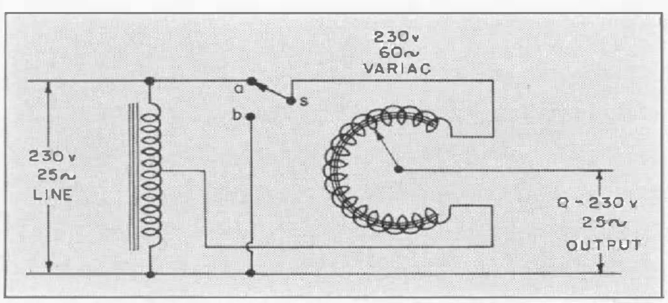


FIGURE 2. Circuit for handling more power than is possible with that of Figure 1.

When it is desired to control a 230-volt line at 25 cycles, auxiliary transformers can be used.

Figure 1 illustrates the use of two auxiliary transformers, one for stepping down the input voltage to the Variac and the other for stepping up the output voltage from the Variac. Figure 2 illustrates a method for obtaining nearly twice the power output obtainable in Figure 1. With the switch "S" in position "b" the output may be controlled from 0-115 volts, while in position "a" control is from 115 to 230 volts. If the switching is done while the brush is at the end connected to the center tap of the auxiliary transformer, there will be no discontinuity in the output voltage, since the Variac is then effectively out of circuit.

— S. A. BUCKINGHAM

Type	*Load Rating*	*Input Voltage*	*Output Voltage*	*Output Current* Rated	*Output Current* Maximum
200-CUH 200-CMH	290 va	115 v	0-135 v	5 a	7.5 a
100-R	1 kva	115 v	0-135 v	9 a	9 a
50-B	3.5 kva	115 v	0-135 v	20 a	31 a